ADDITIONAL TOPICS CHAPTER 8

A graphical approach to long-run capital and labour decisions

In the short run a firm can adjust some of its inputs, such as labour, but it cannot adjust other inputs, such as capital. In the long run, on the other hand, the firm can adjust all its inputs to production, capital as well as labour. When making decisions for the long run, the firm is free to choose any combination of labour and capital. The firm's choice will depend on the relative costs of the inputs and their relative marginal products. If the cost of labour increases, for example, the firm will utilise less labour and more capital.

In this text, we focus on the firm's choice between labour and capital in the long run. We introduce a graphical device similar to the indifference curves used to describe consumer choice. We use this graphical device to show exactly how a firm's choice between labour and capital depends on the relative price and the relative marginal productivities of labour and capital.

Adjusting the mix of capital and labour

Consider an example of a firm with two inputs to production: capital and labour. Table 8A.1 shows the possible combinations of inputs available to firms. For example, if the firm has two units of capital and uses 24 hours of labour, it can produce three units of output. The hypothetical numbers in table 8A.1 could represent a wide variety of firms producing different types of products. Table 8A.1 could refer to many other firms with capital consisting of computers, machine tools, telephones or pizza ovens; to allow for all these

TABLE 8A.1

Production with four levels of capital

QUANTITY PRODUCED	LABOUR INPUT (HOURS)			
	WITH 1 UNIT OF CAPITAL	WITH 2 UNITS OF CAPITAL	WITH 3 UNITS OF CAPITAL	WITH 4 UNITS OF CAPITAL
0	0	0	0	0
1	15	9	6	5
2	27	17	12	10
3	37	24	17	13
4	48	32	22	18
5	60	41	29	23
6	74	51	36	29
7	90	62	43	35
8	109	74	52	41
9	134	87	61	49
10	166	101	71	57
11	216	128	90	72
12	290	170	119	95
13	400	270	189	151
14	—	400	300	220
15	—	—	425	300
16	—	—	—	430

Note: The omitted entries in the table represent quantities of production that cannot be achieved without more capital.

possibilities, we therefore refer to capital as a 'unit' of capital.

The information in table 8A.1 can be represented graphically, as shown in figure 8A.1. Each column is plotted with labour input on the horizontal axis and the quantity produced on the vertical axis. Each column represents a production function for a given level of capital. Note how higher levels of capital increase the amount that can be produced with a given amount of labour. In other words, as we add more capital, the relationship between labour and output shifts up.

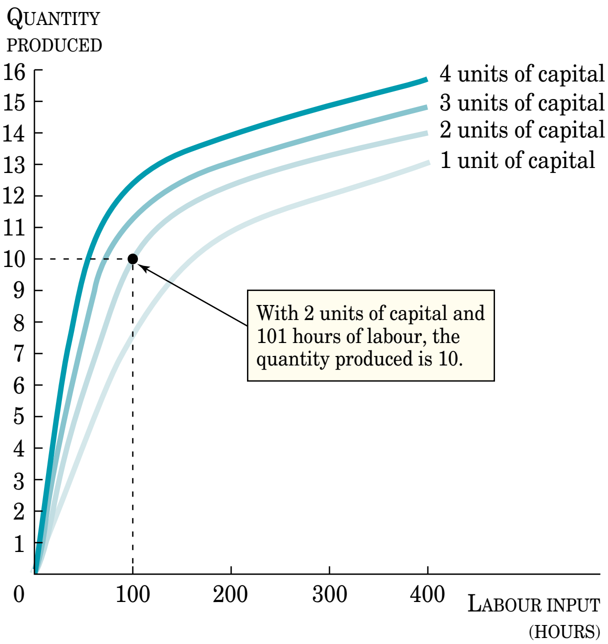

FIGURE 8A.1

The production function with four levels of capital

As the amount of labour input increases, so does the amount of output. Each curve corresponds to a different level of capital. Higher curves represent higher capital. The points on these four curves are obtained from the four columns of table 8A.1.

The information in table 8A.1 and figure 8A.1 can be displayed in another graph, figure 8A.2, which provides a visual picture of how labour and capital jointly help a firm produce its product. Figure 8A.2 puts capital on the vertical axis and labour on the horizontal axis. We represent the quantity produced in figure 8A.2 by writing a number in a circle equal to the amount produced with each amount of labour and capital. For example, with one unit of capital and 60 hours of labour, the firm can produce five units of output, according to table 8A.1. Thus, we write the number '5' at the point in figure 8A.2 that represents labour input equal to 60 and capital input equal to one.

Isoquants

Observe in figure 8A.2 how the same amount of output can be produced using different combinations of capital and labour. We illustrate this in the figure by connecting points with the same quantity by a curved line. Each curve gives the combinations of labour and capital that produce the same quantity of output. The curves in figure 8A.2 are called *isoquants*, where 'iso' means 'the same' and 'quant' stands for 'quantity produced'. Thus, an isoquant is a curve that shows all the possible combinations of labour and capital that result in the same quantity of production. Isoquants convey a lot of information visually. Higher isoquants — those up and to the right — represent higher levels of output. Each isoquant slopes down because as capital input declines, labour input must increase if the quantity produced is to remain the same. The slope of the isoquants tells us how much labour must be substituted for capital (or vice versa) to leave production unchanged. Thus, the isoquants are good for studying how firms substitute one input for another when the prices of the inputs change. The slope of the isoquant is called the rate of technical substitution, because it tells us how much capital needs to be substituted for labour to give the same amount of production when labour is reduced by one unit.

Remember that the points in figure 8A.2 do not display any information not in table 8A.1 or figure 8A.1. The same information appears in a different and convenient way.

Isocost lines

A firm's total costs can also be shown on a diagram like figure 8A.2. In considering the choice between capital and labour, the firm needs to consider the price of both. Suppose that labour costs \$10 per hour and capital costs \$300 per unit. Then if the firm uses one unit of capital and 150 hours of labour, its total costs will be $1 \times \$300 + 150 \times \$10 = \$1800$. For the same total cost, the firm can pay for other combinations of labour and capital. For example, two units of capital and 120 hours of labour also cost \$1800. Other combinations are as follows:

HOURS OF LABOUR	UNITS OF CAPITAL	TOTAL COST
180	0	$180 \times \$10 + 0 \times \$300 = \$1800$
150	1	$150 \times \$10 + 1 \times \$300 = \$1800$
120	2	$120 \times \$10 + 2 \times \$300 = \$1800$
90	3	$90 \times \$10 + 3 \times \$300 = \$1800$
60	4	$60 \times \$10 + 4 \times \$300 = \$1800$
30	5	$30 \times \$10 + 5 \times \$300 = \$1800$
0	6	$0 \times \$10 + 6 \times \$300 = \$1800$

In other words, the $1800 can be spent on any of these combinations of labour and capital. With $1800, the firm can use six units of capital, but that would not permit the firm to hire any workers.

These different combinations of labour and capital that have a total cost of $1800 are plotted in figure 8A.3. Each combination of labour and capital in the table is plotted and the points are connected by a line. The line is called an isocost line. An isocost line shows the combinations of capital and labour that have the same total costs.

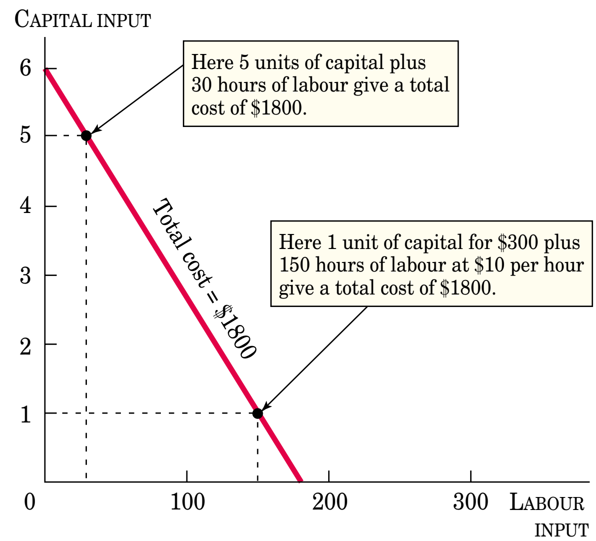

FIGURE 8A.3

An isocost line

Each isocost line shows all the combinations of labour and capital that give the same total costs. In this case, the price of capital is $300 per unit and the price of labour is $10 per hour. Total costs are $1800. For example, if one unit of capital is employed and 150 hours of labour are employed, total costs are $\$1800 = (1 \times \$300) + (150 \times \$10)$.

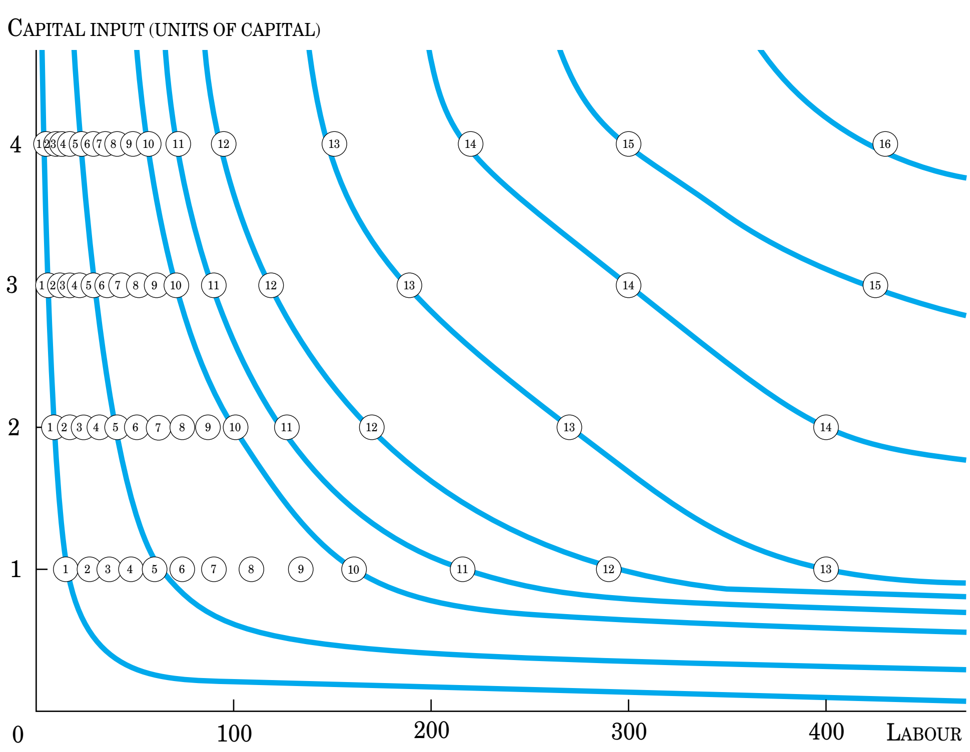

FIGURE 8A.2

Isoquants

The number in each circled point gives the quantity produced for the amount of labour and capital on the axes. The lines connecting equal quantities are called isoquants.

The position of the isocost line depends on the amount of total costs. Higher total costs are represented by higher isocost lines. This is shown in figure 8A.4. Observe that the isocost line for total costs of $2100 is above the one for $1800, but has the same slope.

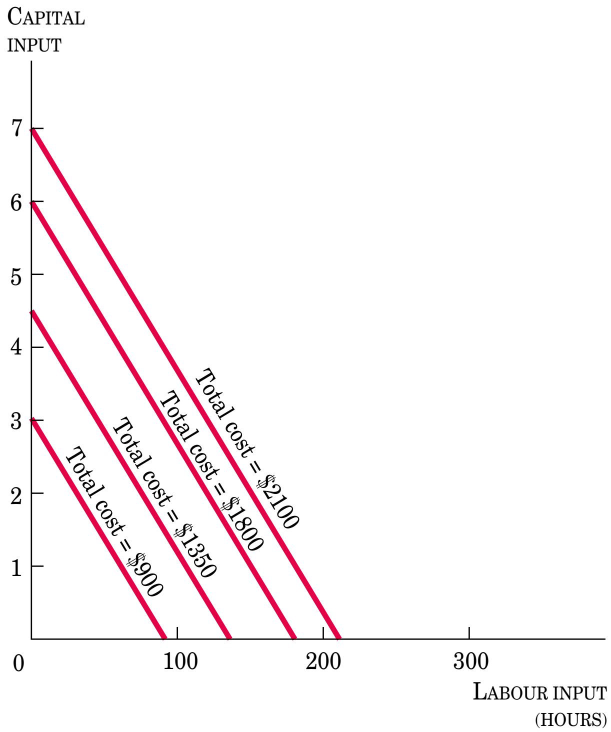

FIGURE 8A.4

Several isocost lines with different total costs

Isocost lines with higher total costs are above and to the right of those with lower total costs. All the isocost lines in this diagram have a capital cost of $300 per unit and a labour cost of $10 per hour.

The slope of the isocost line depends on the ratio of the price of labour to the price of capital. In particular, the slope equals −1 times the ratio of the price of labour to the price of capital. This is illustrated in figure 8A.5 for the case where total costs equal $1800. If the price of labour falls from $10 to $6, then the isocost line gets flatter. Thus, if the hourly wage were $6 instead of $10, the firm would be able to pay for 250 hours of work and one unit of capital, compared with only 150 hours and one unit of capital, and still have a total cost of $1800. Thus, as the price of labour (the wage) falls relative to the price of capital, the isocost line gets flatter.

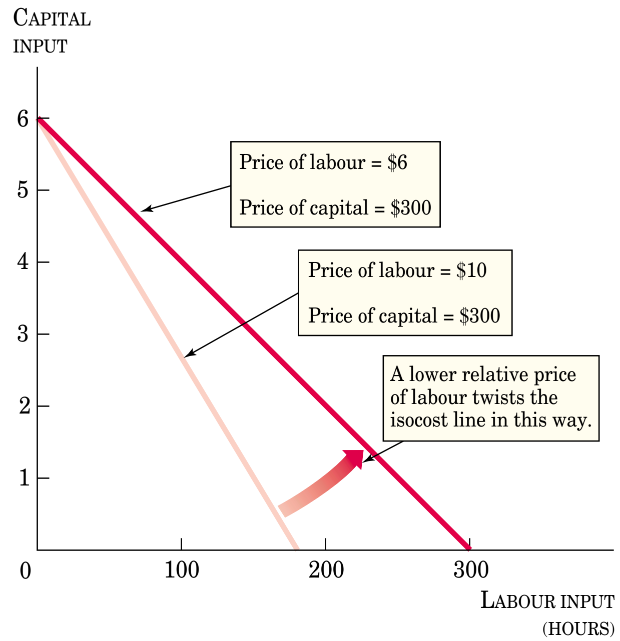

FIGURE 8A.5

Effect of a change in relative prices on the isocost line

When the price of labour falls relative to the price of capital, the isocost line gets flatter, as in this diagram. In this case the price of labour falls from $10 per hour to $6 per hour while the price of capital remains at $300 per unit. Total costs remain equal to $1800 in this case.

Minimising costs for a given quantity

The isoquant and isocost lines can be used to determine the least-cost combination of capital and labour for any given quantity of production. Figure 8A.6 shows how. In figure 8A.6 we show three isocost lines, along with an isoquant representing 11 units of output. For the isocost lines, the price of labour is $10 and the price of capital is $300. The point where the isocost line just touches the isoquant is a *tangency point*. It is labelled *A*.

Point *A* is where the firm minimises the cost of producing 11 units of output. To see this, suppose you are at point *A* and you move to the left and up along the same isoquant to point *B*. This means that the firm increases capital and decreases hours of labour, keeping quantity produced constant at 11 units; that is, the firm would substitute capital for labour. But such a substitution would increase the firm's costs, as shown in the figure. The payment for the extra capital will be greater than the saving from reduced labour. Thus, moving along the isoquant from *A* to *B* would increase the total costs to the firm.

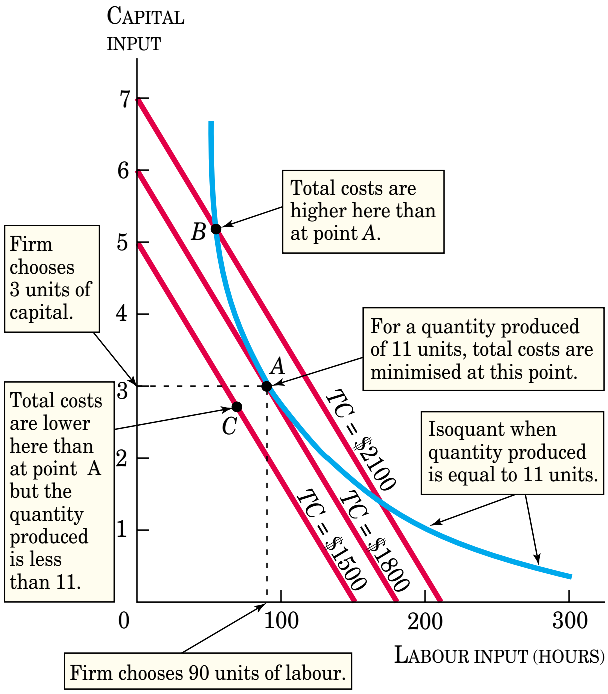

FIGURE 8A.6

Choosing capital and labour to minimise total costs

The diagram illustrates how a firm chooses a mix of labour and capital to minimise total costs for a given level of output. Here the given level of output is 11 units, as shown by the single isoquant. Total costs are minimised by choosing a combination of labour and capital given by the tangency (point *A*) between the isocost line and the isoquant. Any other point on the isoquant would have the same quantity but higher total costs.

A similar reasoning applies to moving from point *A* to point *C*. The firm uses fewer labour hours and less capital at point *C*, so that total costs are lower than at point *A*. But at point *C* it does not have enough inputs to produce 11 units of output. Thus, point *A* is the lowest-cost point at which the firm can produce 11 units of output. It is the point at which the lowest isocost line is touching.

The rate of technical substitution of capital for labour and the ratio of the price of labour to the price of capital coincide at point *A*, because the slope of the isoquant and the isocost line are equal at point *A*. When the rate of technical substitution differs from the input price ratio, the firm is not minimising its costs. (Observe how isoquants are analogous to indifference curves, and the isocost lines are analogous to the budget line.)

A change in the relative price of labour

Now we show how isoquants and isocost lines can be used to predict how a firm will adjust its mix of inputs when there is a change in input prices. For example, suppose that the hourly wage falls from \$10 to \$6 and the price of capital rises from \$300 to \$600. That is, labour becomes cheaper relative to capital. Originally, the ratio of the price of labour to capital was 10/300 = 0.033; now it is 6/600 = 0.010. This is a big reduction, and we would expect the firm to adjust by changing capital and labour input. Figure 8A.7 shows how it would adjust the mix of capital and labour for a given quantity of output.

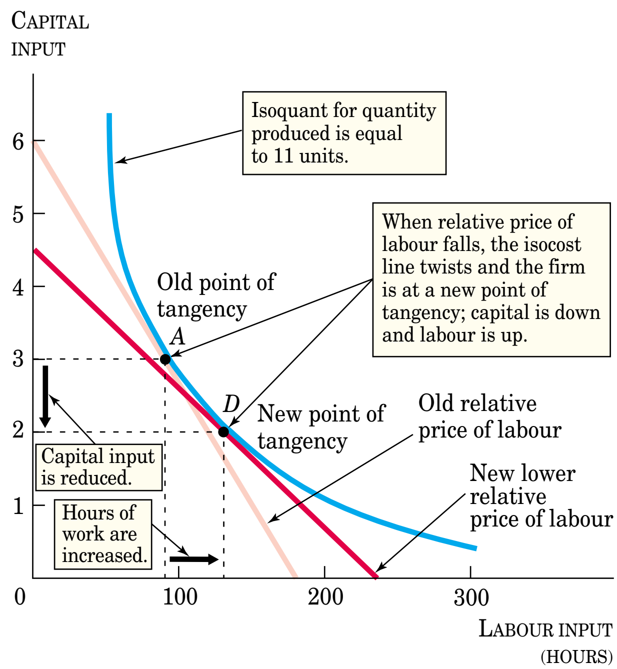

FIGURE 8A.7

Effect of a lower price of labour relative to capital

The dark red isocost line has a lower price of labour relative to capital than the light red line. Hence, the amount of capital used by the firm decreases from three to two, and the amount of labour rises from 90 hours to 130 hours.

Figure 8A.7 keeps the isoquant fixed but includes a new isocost line that reflects the lower relative price of labour and that is tangent to the isoquant. Since the new isocost line is flatter, the point of tangency with the given isoquant no longer occurs at point *A*, where three units of capital were combined with 90 hours of labour. Now tangency occurs at point *D*, where there is a combination of two units of capital and 130 hours of labour. In other words, the firm has substituted labour for capital when the relative price of labour fell. At the new point *D*, the firm would use one less unit of capital and 40 more hours of labour.

In summary, common sense tells us that the firm will hire more labour and use less capital when the price of labour falls relative to the price of capital. The isoquants and isocost lines confirm this and tell us by exactly how much.