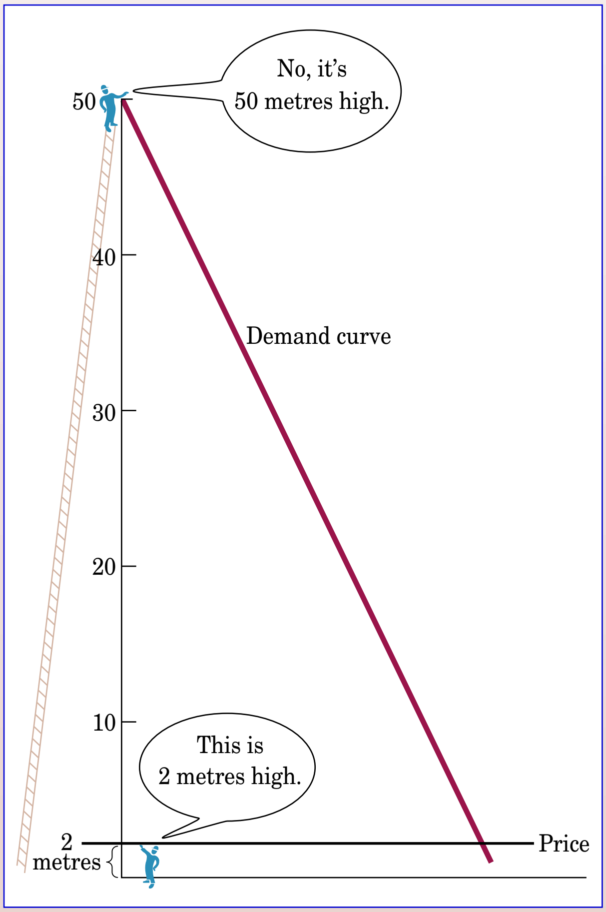
ADDITIONAL TOPICS CHAPTER 5

The origin of consumer surplus

The great British economist Alfred Marshall did the most to develop and communicate the idea of consumer surplus; he put it this way in 1890: 'The price which a person pays for a thing can never exceed, and seldom comes up to that which he would be willing to pay rather than go without it . . . he thus derives from the purchases a surplus of satisfaction . . . It may be called consumer's surplus'.*

However, it was Jules Dupuit, a French engineer, who first came up with the idea. He argued in 1844 that consumer surplus could be used as a way to estimate the benefit of public works, such as roads and bridges. Dupuit also offered a more visual description of consumer surplus: 'If society is paying 500 million for the services rendered by the road, that only proves one thing — that their utility is at least 500 million. But it may be a hundred times or a thousand times greater . . . If you take the [500 million] as the figure . . . you are acting like a man who, wishing to measure the height of a wall in the dark and finding that he cannot reach the top with this raised arm says: "This wall is two metres high, for if it were not, my hand would reach above it". In daylight and equipped with a ladder . . . our alleged two-metre wall is fifty metres high.'†



* Marshall, A. 1920, *Principles of Economics*, 8th ed., Macmillan, New York, p. 124.

† English translation of Dupuit, J. 'De la Mesure de l'Utilité des Travaux Publics', translated and reprinted in K. J. Arrow and T. Skitovsky, eds. 1969, *Readings in Welfare Economics* Irwin, Homewood, Ill.

ADDITIONAL TOPICS CHAPTER 5

Budget lines and indifference curves

The model of consumer behaviour in which utility is maximised subject to a budget constraint can be illustrated with a diagram, as described in this appendix.

We consider a single consumer deciding how much of two items to buy. Let one of the items be X and the other be Y. In Part A we show that the consumer budget constraint can be represented by a budget line, and then in Part B we show that the consumer's preferences can be represented by indifference curves.

Part A: The budget line

Suppose that the consumer has \$20 to spend on X and Y, and suppose that the price of X is \$2 per unit and price of Y is \$4 per unit. How much of X and Y can the consumer buy? If the consumer spends all \$20 on Y, then five units of Y and no units of X are consumed. If the consumer buys four units of Y at \$4 per unit, then \$16 will be spent on Y and the remaining \$4 can be spent buying two units of X. These and several other amounts of X and Y that can be bought with \$20 are shown in the following table.

UNITS OF Y	UNITS OF X	EXPENDITURES
5	0	$5 \times \$4 + 0 \times \$2 = \$20$
4	2	$4 \times \$4 + 2 \times \$2 = \$20$
3	4	$3 \times \$4 + 4 \times \$2 = \$20$
2	6	$2 \times \$4 + 6 \times \$2 = \$20$
1	8	$1 \times \$4 + 8 \times \$2 = \$20$
0	10	$0 \times \$4 + 10 \times \$2 = \$20$

These combinations represent the maximum amounts that can be purchased with \$20. Note how the amounts are inversely related; as more is spent on X, less must be spent on Y. This inverse relationship is shown graphically in figure 5A.1. We put units of Y on the vertical axis and units of X on the horizontal axis, and then plot the pairs of points from the table. The points are then connected with a line. The points trace a downward-sloping line starting in the upper left at $X = 0$ and $Y = 5$ and ending on the right with $X = 10$ and $Y = 0$. All the other combinations of X and Y in the table, such as $X = 4$ and $Y = 3$, are shown on the line. If it is possible to consume fractions of X and Y, then all the points on the line between the plotted points can also be purchased with the \$20. (For example, 2.5 units of Y and five units of X would cost \$20: $2.5 \times \$4 + 5 \times \$2 = \$20$.) Because all these pairs of X and Y on this line can be purchased with a \$20 budget, we call it the budget line. The consumer is constrained to buy combinations of X and Y either on or below the budget line. Amounts of X and Y consumed below the budget line cost less than \$20. Points above the line require more than \$20 and are not feasible.

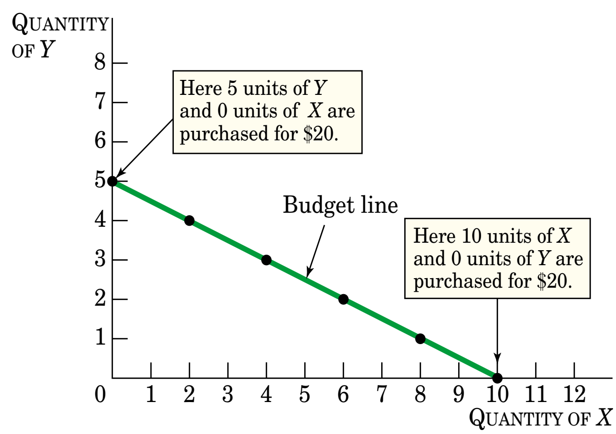


FIGURE 5A.1

Budget line for a consumer

The line shows how much a consumer with \$20 can consume of quantity X at a price of \$2 per unit and quantity Y at \$4 per unit. If \$20 is spent on Y and nothing on X, then five units of Y can be purchased, as shown on the vertical axis. If \$20 is spent on X and nothing on Y, then 10 units of X can be purchased. Other combinations are shown on the line.

The budget line will shift out if the consumer has more to spend, as shown in figure 5A.2. For example, if the consumer has \$24 rather than \$20, then the budget line will shift up by one unit because the extra \$4 permits the consumer to buy one more unit of Y. Alternatively, we could say that the budget line shifts to the right by two units in this case because the consumer can buy two more units of X with \$4 more.

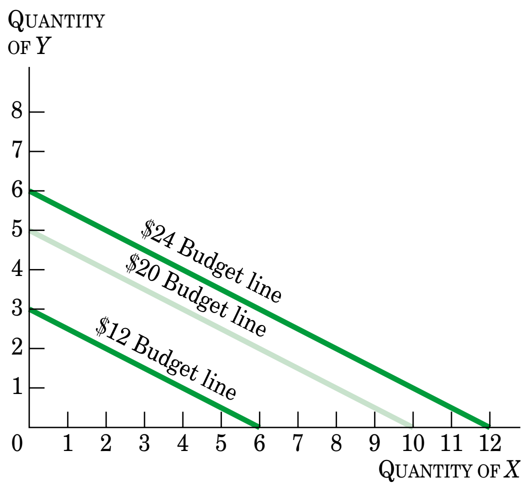


FIGURE 5A.2

Effect of a change in income on the budget line

If the consumer has more to spend, then the budget line is farther out. If the consumer has less to spend, then the budget line is farther in. Here a higher and lower budget line are compared with the $20 budget line in figure 5A.1.

The steepness of the budget line depends on the prices of *X* and *Y*. In particular, the slope of the budget line is equal to −1 times the ratio of the price of *X* to the price of *Y*. That is, slope = $-[P_X / P_Y]$, which is $-\frac{1}{2}$ in this example. Why is the slope determined by the price ratio? Recall that the slope is the change in *Y* divided by the change in *X*. Along the budget line as *X* is increased by one unit, *Y* must fall by half a unit: buying one more unit of *X* costs $2 and requires selling half a unit of *Y*. Thus, the slope is $-\frac{1}{2}$.

In order to derive the demand curve for *X*, we need to find out what happens when the price of *X* changes. What happens to the budget line when the price of *X* increases from $2 to $4, for example? The budget line twists down, as shown in figure 5A.3. You can show this by creating a new table with pairs of *X* and *Y* that can be purchased with $20 at the new price and then plotting the points. The intuitive rationale for the twist is that the slope steepens to $-[P_X / P_Y]$ = −$4/$4 = −1, and the position of $X = 0$ and $Y = 5$ on the vertical axis does not change because five units of *X* can still be purchased.

To summarise, we have shown how a budget line can be used to represent the budget constraint for the consumer; now consider how we represent the consumer's preferences.

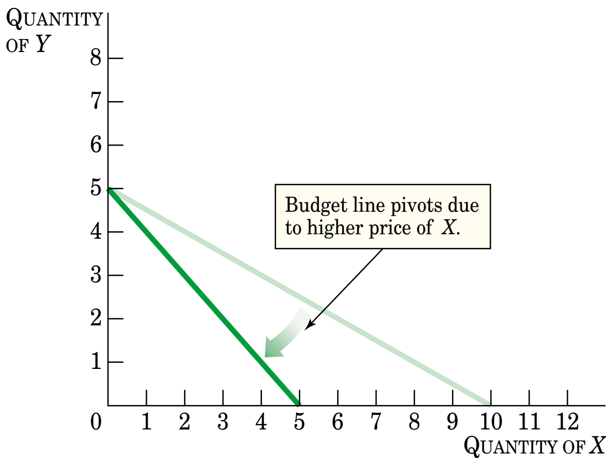


FIGURE 5A.3

Effect of a higher price of X on the budget line

The budget line pivots if the price of *X* changes. Here the price of *X* rises from $2 to $4 and the budget line twists down.

Part B: the indifference curve

Recall from the chapter that utility is an indication of how a consumer prefers one item in comparison with another. If the level of utility is the same for two combinations of *X* and *Y*, then the consumer is *indifferent* between the two combinations. Suppose that the utility is the same for the combinations of *X* and *Y* that appear in the table below.

UNITS OF *Y*	UNITS OF *X*
6	1
4	2
2	6
1	12

The consumer is indifferent between any of these combinations. Observe how these amounts are inversely related. As consumption of *Y* declines, the consumer must be compensated with more *X* if the level of utility is not to decline.

We can plot these different amounts on the same type of graph we used for the budget line, as shown in figure 5A.4. The consumer is indifferent between all four points. We have connected the points with a curve to represent other combinations of *X* and *Y* about which the consumer is indifferent. The curve is called an indifference curve because the consumer is indifferent between all the points on the curve.

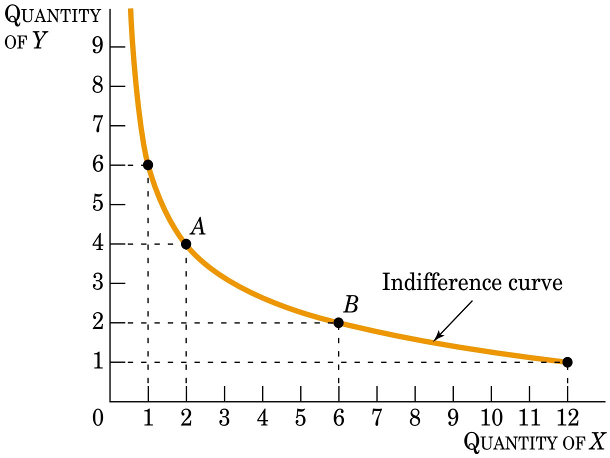


FIGURE 5A.4

An indifference curve for a consumer

The consumer is indifferent to *A* or *B* or any other point on an indifference curve. For example, the consumer is indifferent between consuming four of *Y* and two of *X* or two of *Y* and six of *X*.

The indifference curve slopes downwards from left to right and is bowed in at the origin. Note how the indifference curve is steep when a small amount of *X* is consumed and flat when a small amount of *Y* is consumed. This curvature is due to declining marginal utility. When the consumer is consuming only a little bit of one good, a large amount of the other good is required for compensation of a reduction in the first good.

The slope of the indifference curve is equal to –1 times the ratio of the marginal utility of *X* to the marginal utility of *Y*; that is, slope = $-[MU_X / MU_Y]$. The reason is that utility is the same for all points on an indifference curve. In other words, the decline in utility as *X* falls $(-MU_X \times \Delta X)$ must equal the increase in utility as *Y* rises $(MU_Y \times \Delta Y)$. Thus, $(MU_X \times \Delta X) = -(MU_Y \times \Delta Y)$, which implies that $\Delta Y/\Delta X = -MU_X/MU_Y$, which is the slope of the indifference curve.

We can represent higher levels of utility or more preferred combinations of *X* and *Y* by higher indifference curves, as shown in figure 5A.5. Any point on a higher indifference curve is preferred to any point on a lower indifference curve.

Getting to the highest indifference curve given the budget line

Now we can combine the budget line and the indifference curve on the same diagram to illustrate the model of consumer behaviour. Utility maximisation subject to the budget constraint means getting to the highest possible indifference curve without going above the budget line. The process is shown in figure 5A.6. The budget line from figure 5A.1 and the indifference curves from figure 5A.5 are shown in the diagram.

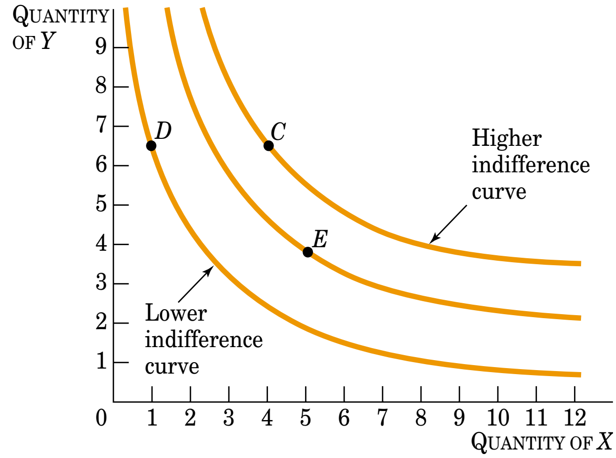


FIGURE 5A.5

Higher and lower indifference curves

Amounts of *X* and *Y* on indifference curves that are higher are preferred to amounts lower on the indifference curves. The combination at *D* is the least preferred and the combination at *C* is the most preferred of the three combinations.

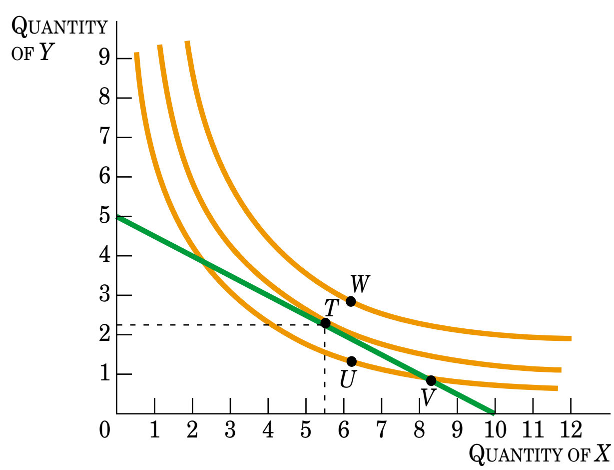


FIGURE 5A.6

The best choice for the consumer

When the budget line is tangent to the indifference curve, the consumer cannot do any better. The point of tangency is at point *T*. Compare this with the other points. Point *U* is not the best point because it is inside the budget line. Point *V* is not the best point because there are other points on the budget line that are preferred. Point *W* is preferred to point *T*, but it is not feasible.

The consumer cannot go beyond the budget line, and any point inside the budget line is inferior to points on the budget line. Thus, the combination of *X* and *Y* with the highest utility must be on the budget line. The highest indifference curve with points on the budget line is the one that just touches — is tangent to — the budget line. This occurs at point *T* in figure 5A.6. The tangency point is the highest level of utility the consumer can achieve subject to the budget constraint. It is the combination of *X* and *Y* that the consumer chooses. The diagram shows that, in this example, the consumer buys 2.25 units of *Y* and 5.5 units of *X*.

Effect of a price change on the quantity of *X* demanded

Now suppose the price of *X* increases; then the budget line twists down, as shown in the right-hand panel of figure 5A.7. With the new budget line, the old consumer choice of 2.25 units of *X* and 5.5 units of *Y* is no longer feasible: point *T* is outside the new budget line. The highest level of utility the consumer can now achieve is at point *S* in the lower panel of figure 5A.7. At point *S* the quantity of *X* has declined. Thus, a higher price of *X* has reduced the quantity of *X* demanded.

In the left-hand panel, we show the relationship between the price of *X* and the reduction in the quantity of *X* demanded. The price of *X* is put on the vertical axis and the quantity of *X* demanded is put on the horizontal axis. The lower quantity demanded at the higher price shows the negative slope of the demand curve.

Effect of an income change on demand

We can also examine what happens when the consumer's income changes but the price remains constant. This is illustrated in figure 5A.8, where income declines. The lower income leads to less consumption of *X* and *Y*. In this case, both *X* and *Y* are normal goods because consumption falls when income falls (see chapter 3). If the consumption of *X* increases as the budget curve shifts in, then *X* would be an inferior good.

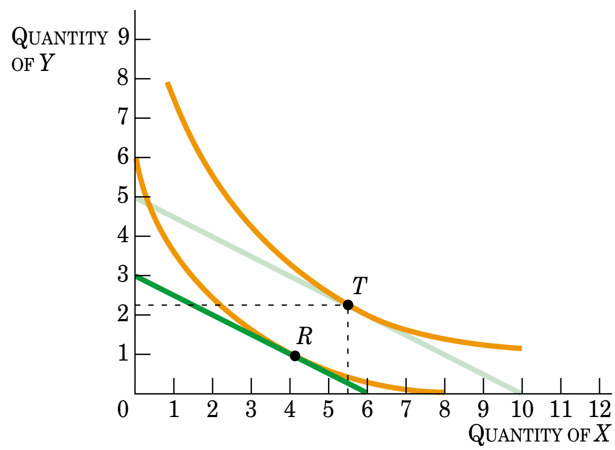


FIGURE 5A.8

A decrease in income

If the consumer's income falls, there is a new point where utility is maximised: the consumer moves from point *T* to point *R*. In this case, consumption of both *X* and *Y* decline. Neither good is an inferior good in this example.

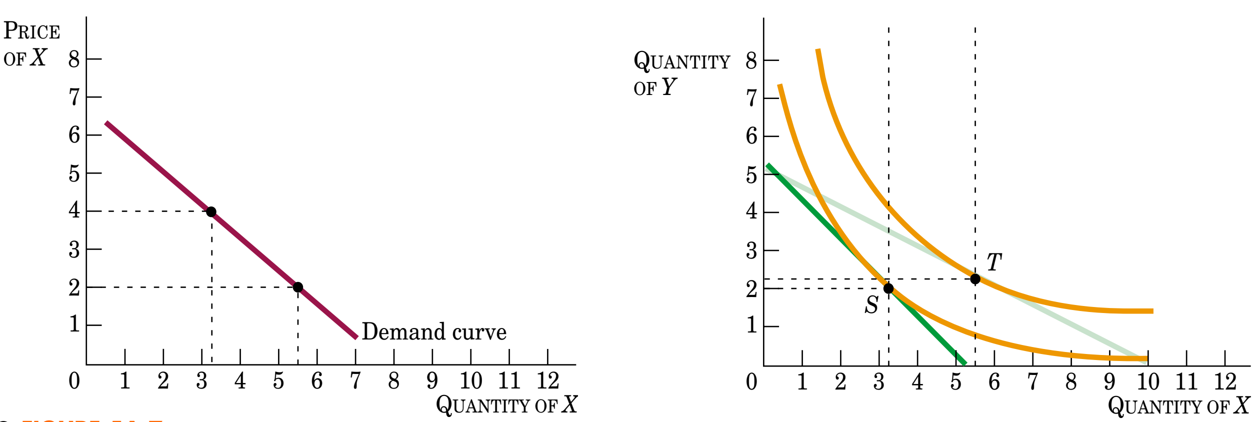


FIGURE 5A.7

An increase in the price of X

If the price of *X* rises, the budget line pivots down and the consumer's choice changes from point *T* to point *S* in the lower panel. The quantity of *X* consumed goes down; the price of *X* and the quantity of *X* are plotted in the top panel, showing the negative relationship between price and quantity demanded.