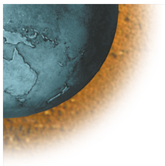

# ADDITIONAL TOPICS CHAPTER 5

## Rules for utility maximisation

The description of utility maximisation using a table as an example is straightforward. One scans the columns searching for the maximum value of utility for which expenditures are equal to or less than the budget constraint. Such a scan and search process is similar to how research economists find the maximum utility in practical applications when there are many different goods, though they are assisted by computers to help find the maximum.

There is another less direct approach to utility maximisation that provides additional insights. The approach makes use of the utility maximising rule, which states that when a consumer maximises utility, the ratio of the marginal utilities of any two goods equals the ratio of the prices of the two goods. Thus, in the case of grapes and bananas:

$$\frac{MU_G}{MU_B} = \frac{P_G}{P_B} \text{ (utility maximising rule)}$$

where $MU_G$ is the marginal utility of grapes, $MU_B$ is the marginal utility of bananas, $P_G$ is the price of grapes and $P_B$ is the price of bananas.

Note how the rule works when the prices are \$1 per kilogram of grapes and \$1 per kilogram of bananas. We already showed that the consumer with an \$8 budget constraint chooses to consume four kilograms of grapes and four kilograms of bananas to maximise utility. At this amount of consumption the marginal utility of grapes is two and the marginal utility of bananas is two, so that the ratio of marginal utilities is one ($2 \div 2 = 1$) and the ratio of the prices is one ($\$1 \div \$1 = 1$). For the case when $P_G = \$2$ and $P_B = \$1$, the price ratio is two; in this case the utility maximisation choice is two kilograms of grapes and four kilograms of bananas, and the ratio of the marginal utilities is two.

To see why the rule works, first imagine that the rule was violated. Suppose that the price ratio is one and that the \$8 is being spent in such a way that the marginal utility ratio was four, as would occur when two kilograms of grapes and five kilograms of bananas are consumed. Then the individual could buy one more kilogram of grapes and one less kilogram of bananas and increase utility. Because the marginal utility ratio is four, the increase in utility from grapes is greater than the loss in utility from one less kilogram of bananas. When the utility maximising rule holds, this situation can't happen. If the ratio of marginal utility was one, then the increased utility from more grapes would exactly equal the decreased utility from fewer bananas. So we know the consumer is maximising utility.

A second way to see why the utility maximising rule works is to write it in an alternative form:

$$\frac{MU_G}{P_G} = \frac{MU_B}{P_B} \text{ (utility maximising rule — alternative form)}$$

In words, this expression states that if the consumer is maximising utility, then the *marginal utility per additional dollar spent on one good must equal the marginal utility per additional dollar spent on another good.* For example, if $P_G = \$2$ and $P_B = \$1$, then $MU_G = 2$ and $MU_B = 1$ for the utility maximisation, and $MU_G / P_G = 2/2 = 1$ with $MU_B / P_B = 1/1 = 1$. The rule holds. If, to the contrary, the rule were violated, then the consumer could change consumption and increase utility. If the marginal utility per additional dollar spent on grapes was four and the marginal utility per additional dollar spent on bananas was only one, the consumer could increase utility by buying more grapes and fewer bananas. This would reduce the marginal utility of grapes and raise the marginal utility of bananas until the two were equal.

Putting the utility maximising rule in this alternative form shows how it can be used to describe many decisions people make every day. For example, suppose you are interested in maximising your university results. Then in balancing your time spent studying for biology versus economics, you will want to make sure that more time spent on economics would not improve your economics grade by more than the time lost studying biology would worsen your biology grade. If spending more time on economics would increase your economics grade by more than it worsens your biology grade, then you should increase the time spent on economics. When the additional minute spent studying economics affects your grade by the same amount that it reduces your biology grade then you are maximising your overall results. By equating the marginal change in your biology grade and your economics grade, you are allocating your time to achieve the highest possible results.

# ADDITIONAL TOPICS CHAPTER 5

## The diamond–water paradox

The consumer behaviour model just discussed can explain a number of otherwise puzzling observations. For example, one of the most quoted passages of Adam Smith's *Wealth of Nations* was the observation that 'Nothing is more useful than water: but it will purchase scarce any thing; scarce any thing can be had in exchange for it. A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it' (Smith, A. 1994 (1776), *Wealth of Nations*, Modern Library Edition, New York, pp. 31–2). Why are diamonds expensive and water cheap even though diamonds are less 'useful' to the world's population than water?

The utility maximising rule helps explain the paradox. The ratio of the price of diamonds to the price of water will be high if the ratio of the marginal utility of diamonds to the marginal utility of water is high. As we saw earlier in the chapter, the marginal utility of something declines the more people consume of it. Thus, water has relatively low marginal utility because, with water being so plentiful, people consume much of it every day. The marginal utility is low even though the total utility from water consumption in the world is very high. On the other hand, diamonds have a high marginal utility because, with diamonds being so scarce, people consume relatively little of them. The marginal utility of diamonds is high even though the total utility of diamonds may be low. With the marginal utility of water low compared with the marginal utility of diamonds, the price of water must be low compared with the price of diamonds. Thus, by distinguishing utility from marginal utility, the latter concept was not known to Smith, we can explain the diamond–water paradox.

The diamond–water paradox is important because it is an extreme example of a very common occurrence.