

ADDITIONAL TOPICS CHAPTER 9

Deriving the growth accounting formula

Figure 9A.1 shows the relationship between productivity (Y/L) and capital per hour of work (K/L). Because of diminishing returns to capital, the line is curved: as capital per hour of work increases, the increased productivity that comes from additional capital per hour diminishes.

The curve in figure 9A.1 is called a productivity curve; it can be represented in symbols as $(Y/L) = f(K/L)$, or productivity is a function of capital per hour of work.

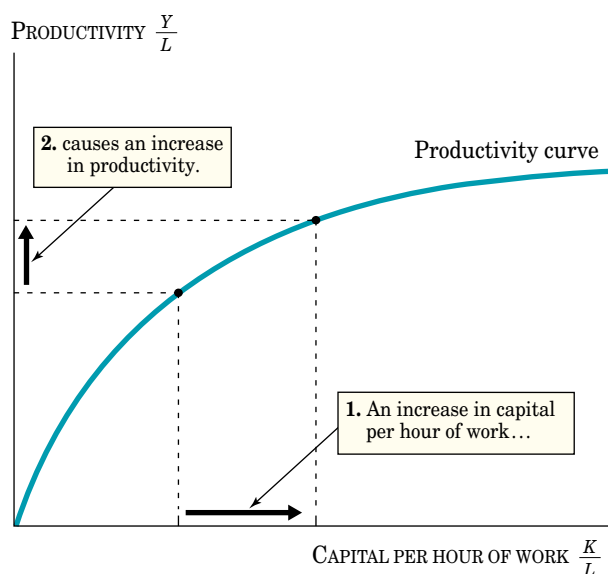


FIGURE 9A.1

Productivity curve

Productivity, or output per hour of work, is shown to increase with the amount of capital that workers have, as measured by capital per hour of work. The productivity curve gets flatter as output per hour of work increases because of diminishing returns to capital.

An upward shift in the productivity function due to an increase in technological change is shown in figure 9A.2. For example, with capital per hour constant at point A in the figure, more technology leads to more productivity.

We can derive the growth accounting formula using this graph in much the same way as Robert

Solow did in 1957. Productivity increases in the economy are due to a combination of *movements along* the productivity function, because of more capital per hour, and of *shifts* of the productivity function, because of technological change.

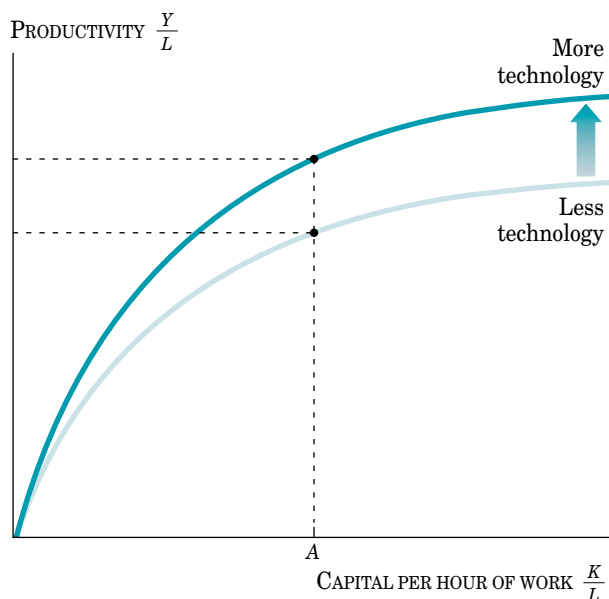


FIGURE 9A.2

A shift in the productivity curve due to technology

An increase in technology permits an increase in productivity even if there is no change in capital per hour of work. For example, if capital per hour of work stays at A, productivity increases when the productivity curve shifts up.

In figure 9A.3, productivity and capital per hour in two different years (year 1 and year 2) are shown. These could be 1999 and 2000 or any other two years. In this example, the growth rate of productivity is given by the increase in productivity (C minus A) divided by the initial level of productivity (A), or $(C - A)/A$. (The definition of the growth rate of a variable is the change divided by the initial level.) Observe in figure 9A.3 how the increase in productivity can be divided into the part due to technology (B minus A) and the part due to higher capital per worker hour (C minus B).

Thus, we have:

$$\underbrace{\left| \frac{(C-A)}{A} \right|}_{\text{growth rate of productivity}} = \underbrace{\left| \frac{(B-A)}{A} \right|}_{\text{growth rate of technology}} + \underbrace{\left| \frac{(C-B)}{A} \right|}_{\text{term related to capital per hour}}$$

which is close to the growth accounting formula. To finish, we need to examine the second term on the right $\left[\frac{(C-B)}{A} \right]$. How does this term relate to capital per hour of work? The boxes in figure 9A.3 show that $C - B$ equals the *change* in capital per hour of work $\Delta(K/L)$ times the *slope* of the productivity function. (The slope times the change along the horizontal axis gives the changes along the vertical axis.) Let the symbol r be the slope, which measures how much additional capital increases output. Thus, $(C - B)/A$ is given by:

$$\frac{\Delta(K/L)r}{Y/L} = \frac{\Delta(K/L)}{(K/L)} r(K/Y)$$

The expression on the right is obtained by multiplying the numerator and the denominator of the expression on the left by (K/Y) . Now the expression on the right is simply the growth rate of capital per hour times $r(K/Y)$. The amount of income paid to capital is r times K if capital is paid according to how much additional capital increases output. Aggregate income is given by Y . Thus, the term $r(K/Y)$ is the share of capital income in aggregate income. As explained in the text, $r(K/Y)$ is approximately $\frac{1}{3}$. Thus, the expression $(C - B)/A$ is the growth rate of capital per hour of work times the share of capital income in aggregate income. Thus, the growth accounting formula is derived.

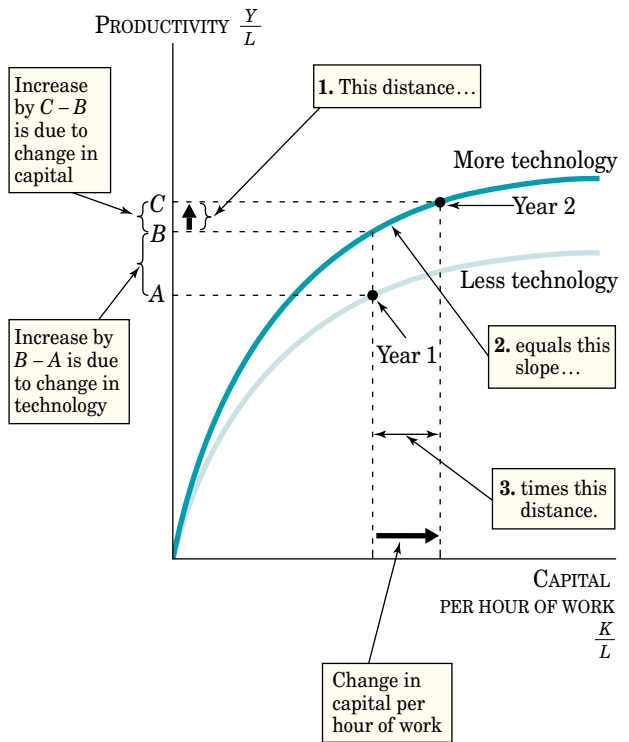


FIGURE 9A.3

Growth accounting with capital per hour and technology increasing

Here a shift in the productivity curve due to more capital per hour of work and a movement along the productivity curve due to more technology are combined. Productivity increases. The part of the increase due to capital and the part due to technological change are shown in the diagram.